MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

TMA1101 – CALCULUS

(All sections / Groups)

25 FEBRUARY 2017
2:30 p.m. – 4:30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of six pages with **FIVE** questions.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided.
4. **No calculators are allowed**.
5. **You are required to write proper steps.**

ANSWER ALL QUESTIONS.

QUESTION 1 [10 marks]

1 (a) Find the following limits.

[***You must show at least one intermediate step where*** $\lim_{x\to c}$ ***is still needed.***]

(i) $\lim_{x\to 1} \dfrac{x^2-1}{(x-1)(x^6+1)}$

(ii) $\lim_{x\to 0} \dfrac{x^2}{1+\sin x-\cos x}$ [2 marks]

(b) Given $f(x)=\begin{cases} 2x+4 & \text{if } 1<x<2 \\ 6 & \text{if } x=2 \\ 2x^2 & \text{if } x>2 \end{cases}$

(i) Determine $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$.

[***For this part, you must show at least at least one intermediate step where*** $\lim_{x\to 2^-}$ ***or*** $\lim_{x\to 2^+}$ ***is still needed.***]

(ii) Determine $\lim_{x\to 2} f(x)$.

(iii) Is the function f continuous at 2? Give the reason for your answer.

[4 marks]

(c) (i) State the Intermediate Value Theorem (i.e., the full statement including the hypothesis and the conclusion).

(ii) Show that there is a root of the equation $x^4+3x-3=0$ in the interval $[-1,1]$.

You must write proper steps to arrive at the conclusion; just writing some calculations would not be enough.

[4 marks]

Continued .....

QUESTION 2 [10 marks]

(a) Use the formal definition of derivative to find $f'(-1)$ when $f(x) = x^2 + 3x$.

You are reminded to write proper steps. [2.5 marks]

(b) Find $\frac{dy}{dx}$ with y as given.

[Use the product rule or the quotient rule for differentiation; show proper steps.]

(i) $y = e^{-x} \sin x$

(ii) $y = \frac{\ln x}{x}$ [3 marks]

(c) The point (1, 2) lies on the curve $2y^2 + 2xy - x^2 = 11$.

Use implicit differentiation to obtain $\frac{dy}{dx}$ **in terms of x and y.**

Then determine the gradient of the tangent to the curve $2y^2 + 2xy - x^2 = 11$ at the point (1, 2).

[4.5 marks]

Continued .....

QUESTION 3 [10 marks]

(a) (i) Determine the values of A and B in the following partial fraction decomposition.

$$\frac{5x}{x^2+x-6} = \frac{A}{x+3} + \frac{B}{x-2}$$

(ii) Then evaluate $\int \frac{5x}{x^2+x-6} dx$ [3 marks]

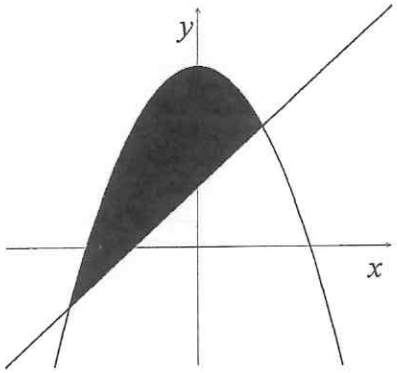
(b) (i) Use $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$ and $\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$ to find the values of A and B which make the equation

$$\sin x \cos 3x = A\sin 2x + B\sin 4x$$

an identity.

(ii) Evaluate $\int_0^{\frac{\pi}{4}} \sin x \cos 3x \, dx$ [3.5 marks]

(c) The figure shows a region bounded by the parabola $y = 3 - x^2$ and the straight line $y = x + 1$.



(i) Determine the x-coordinates of the points of intersection between the parabola and the straight line.

(ii) Write down a definite integral that can be used to find the area of this region and proceed to find the area.

[3.5 marks]

Continued .....

QUESTION 4 [10 marks]

(a) For what values of p is the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ convergent?

Compare $\sum_{n=1}^{\infty} \frac{1}{n^2+3}$ with a particular p-series (i.e., suggest a suitable value for p) to determine if $\sum_{n=1}^{\infty} \frac{1}{n^2+3}$ is convergent. [2 marks]

(b) Let $a_n = \frac{n}{3^n}$. Determine $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

Then use the **ratio test** to determine whether the infinite series $\sum_{n=1}^{\infty} \frac{n}{3^n}$ is convergent. [2 marks]

(c) Find the Maclaurin polynomial of order 3 for $f(x) = \frac{1}{x+2}$.

[You can use fractions for the coefficients; there is no need to convert to decimals.] [3 marks]

(d) A periodic function f of period 2π is defined as

$$f(x) = \begin{cases} 0, & 0 \le x < \pi \\ 3, & -\pi \le x < 0 \end{cases}$$

(i) Sketch a graph of f in the interval $-2\pi < x < 2\pi$.

(ii) The **Fourier series** of f has the form

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx).$$

Find the value of a_3. [3 marks]

Continued .....

QUESTION 5 [10 marks]

(a) Find the partial derivatives $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$ for the function

$$F(x,y) = x^2 + xy - e^y \ln x .$$ [1 mark]

(b) Solve the first order **separable equation** $\frac{dy}{dx} = \frac{3x^2}{y^2}$

subject to the initial condition $y(0) = 1$.

You may leave your answer in implicit form. [2.5 marks]

(c) You are told that e^{-2x} is an integrating factor for the first order linear equation

$\frac{dy}{dx} - 2y = 3e^{2x}$.

Solve the equation and give your solution in explicit form. [2 marks]

(d) (i) Find the roots of the **characteristic equation** of the homogeneous differential equation $y''-2y'-3y = 0$.

Then write down the general solution y_h of this homogeneous differential equation.

(ii) If $y = ax + b$ is a **particular solution** of the differential equation

$$y''-2y'-3y = 3x-1,$$

determine the values of a and b.

(iii) Hence, write down the **general solution** for the differential equation

$y''-2y'-3y = 3x-1$. [4.5 marks]

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